

Modified Gravity Explains Dark Matter?

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We explore a new horizon of modified gravity from the viewpoint of the particle physics. As a concrete example, we take the $F(R)$ gravity to raise a question: can a scalar particle (“scalon”) derived from the $F(R)$ gravity be a dark matter candidate? We place the limit on the form of function $F(R)$ from the constraint on the scalaron as a dark matter. The role of the screening mechanism and compatibility with the dark energy problem are addressed.

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I. INTRODUCTION

Late-time accelerated expansion of the Universe has been confirmed by several independent observations [1–9]. In order to explain the accelerated expansion, it could be inevitable to include the Dark Energy (DE) as the new energy source. In addition to the existence of the DE, the observations of the rotation curve of galaxies and the gravitational lensing indicate the presence of new matters, which do not have the electromagnetic interaction but the gravitational one, so-called the Dark Matter (DM). The origin of such a dark sector involving the DE and DM is still mysterious among fields of the particle physics and astrophysics.

The Λ CDM model provides the simplest way to account for the DE as well as the DM, in which the cosmological constant Λ and the Cold Dark Matter (CDM) are introduced in the framework of the general relativity (GR). This model successfully describes the almost all of the cosmic history. However, it suffers from several theoretical problems: (i) the extremely large discrepancy between the theoretical and observational values of the cosmological constant, which is known as a fine-tuning problem; (ii) the ratio of the DE to the CDM with respect to the current energy density, which is known as a coincidence problem. The Λ CDM does not give us any answer to these two problems, which would imply the necessity of a new paradigm.

It would be the modified gravity theory that can solve those problems, which has been intensively investigated so far. The modification of gravity brings a new degree of freedom, such as a scalar field, which can mimic the role of the cosmological constant (for a review, see [10–17]), so that one can explain the late-time cosmic acceleration without invoking ad hoc introduction of the cosmological constant. Hence, the problem (i) is initially not present. Besides the DE problem, it has been suggested that a new particle derived from the modification of gravity can be a dark matter candidate [18–20]. In this scenario, one might be able to predict the ratio of the DE to the DM regarding the energy density today, and give the answer to the problem (ii).

Of interest is that the DM candidate is naturally in-

troduced merely due to the modification of gravitational theory, without any ad hoc assumptions. More remarkably, this DM possesses a salient feature which cannot be seen either in the Weakly Interacting Massive Particle scenario, or in the axion DM scenario. That is the screening mechanism: the propagation of the DM can be regarded as the fifth force in the astrophysical observation, which is suppressed in the modified gravity. In a similar way, the propagation of the DM in the Standard Model (SM) bulk should be suppressed. Thus, the screening mechanism would trigger the non-trivial effect on the coupling between the DM and the SM particles. Recently, the DM from the modified gravity theory has been discussed in the context of particle physics [21–24]. However, the screening mechanism was not taken into account when the DM couplings with the SM are formulated.

In this paper, we study a DM candidate derived from the $F(R)$ gravity, one of modified gravity theories, taking into account the screening mechanism properly. The $F(R)$ gravity can be expressed as the scalar-tensor theory, which includes the Einstein–Hilbert action and an extra scalar field. This scalar is the DM candidate, which we shall call “scalon”. If we regard the scalaron as a DM, it plays two different roles: it acts as the DE at cosmological scale while it is a DM candidate at smaller scales. This scale-dependent behavior is reflected by the screening mechanism, which is called the chameleon mechanism in the $F(R)$ gravity. Due to the chameleon mechanism, the scalaron becomes heavy around the high density region, and the $F(R)$ gravity can avoid the Solar System constraint.

The key idea we shall propose is that the chameleon mechanism is applied to the microscopic environment: one naively expects that the scalaron becomes heavy even in the SM bulk because the density of the ensemble made of the SM particles, estimated from the macroscopic view, is high enough. However, *it is nontrivial how the chameleon mechanism works for the scalaron interacting with the SM in the framework of the particle physics*, although the chameleon mechanism is well understood on the astrophysical ground.

Based on this idea, we derive the couplings between

the scalaron and the SM particles in the quantum field theory. We evaluate the lifetime of the scalaron by taking into account the chameleon mechanism, to find the upper limit of the scalaron mass. We then discuss the constraint on the form of the $F(R)$ function by the relation between the $F(R)$ function and the scalaron potential.

II. $F(R)$ GRAVITY AND THE WEYL TRANSFORMATION

In this section, we give a brief review of the $F(R)$ gravity, and observe how the scalar field emerges via the Weyl transformation of the metric. We begin with the action of $F(R)$ gravity defined as follows:

$$S = \frac{1}{2\kappa^2} \int d^4x \sqrt{-g} F(R) + S_{\text{Matter}}, \quad (1)$$

where $\kappa^2 = 8\pi G = 1/M_{\text{pl}}^2$ and M_{pl} is the reduced Planck mass. $F(R)$ is a function of the Ricci scalar R : e.g. $F(R) = R$ in general relativity. From the action (1), we obtain the following equation of motion,

$$\begin{aligned} \kappa^2 T_{\mu\nu} = & F_R(R) R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} F(R) \\ & + (g_{\mu\nu} \square - \partial_\mu \partial_\nu) F_R(R), \end{aligned} \quad (2)$$

where $F_R = \partial_R F(R)$ for convenience.

Sharp contrast to the Einstein equation stems from the trace part of Eq. (2): by performing the trace of Eq. (2), we obtain the Klein-Gordon type equation,

$$\square F_R(R) = \frac{1}{3} \kappa^2 T + \frac{1}{3} [2F(R) - F_R(R)R]. \quad (3)$$

Eq. (3) implies that R is dynamical although it is determined by the algebraic relation with the energy-momentum tensor, $R = -T$, in the general relativity. This scalar degree of freedom corresponds to the scalar function $F(R)$, and it appears if and only if $F(R) \neq R$.

Next, we extract the dynamics of the scalar degree of freedom in the $F(R)$ gravity. We first rewrite the action (1) by introducing an auxiliary field A in the following form:

$$S = \frac{1}{2\kappa^2} \int d^4x \sqrt{-g} [F_A(A)R - \{F_A(A)A - F(A)\}]. \quad (4)$$

By the variation of (4) with respect to A , we obtain the equation of motion for A : $F_{AA}(A)(R - A) = 0$, and we find $A = R$ if $F_{RR}(R) \neq 0$ for all R . Substituting $A = R$ into the action (4) again, one can reproduce the original action (1).

The above relation between A and R is consistent with the fact that the Ricci scalar is dynamical in the $F(R)$ gravity as Eq. (3) implies. Therefore, we find that the dynamics of $F(R)$ gravity is equivalent to the dynamics of the general relativity with the non-minimal coupling

to the scalar field A . Note that this newly introduced scalar field brings a modification of gravity, and plays a significant role in cosmology if the mass of the scalar field is as small as the cosmological constant.

Next, we deform the non-minimal coupling between the scalar field A and the Ricci scalar R into the minimal coupling by the Weyl transformation. The Weyl transformation of the metric is defined as $g_{\mu\nu} \rightarrow \tilde{g}_{\mu\nu} = e^{2\sigma(x)} g_{\mu\nu}$. It can be seen as the transformation of frame; the Jordan frame described by the original metric $g_{\mu\nu}$ is transformed to the Einstein frame with $\tilde{g}_{\mu\nu}$. Under the Weyl transformation, the line element in the Einstein frame is

$$\begin{aligned} d\tilde{s}^2 &= \tilde{g}_{\mu\nu} d\tilde{x}^\mu d\tilde{x}^\nu \\ &= e^{2\sigma} g_{\mu\nu} dx^\mu dx^\nu = e^{2\sigma} ds^2. \end{aligned} \quad (5)$$

Here, we note that the Weyl transformation changes the distances between the two points described by the same coordinate system x^μ on the manifold. Thus, the definitions of time and length are different between two frames: after the calculation in the Einstein frame, dimensionful observables in the Jordan frame are evaluated by the scale transformation according to (5). Hereafter, we will use the partial derivative as ∂_μ (with lower index) and the coordinate as x^μ (with upper index), to avoid confusion in raising and lowering the indices by $g_{\mu\nu}$ or $\tilde{g}_{\mu\nu}$.

Under the Weyl transformation, the action (4) is transformed into the following form:

$$\begin{aligned} S = & \frac{1}{2\kappa^2} \int d^4x \sqrt{-\tilde{g}} [F_A(A) e^{-2\sigma} \\ & \times \{ \tilde{R} + 6\tilde{g}^{\mu\nu} \tilde{\nabla}_\mu \partial_\nu \sigma - 6\tilde{g}^{\mu\nu} (\partial_\mu \sigma) (\partial_\nu \sigma) \} \\ & - e^{-4\sigma} \{ F_A(A) A - F(A) \}]. \end{aligned} \quad (6)$$

By defining the Weyl transformation as $e^{2\sigma} \equiv F_A(A)$, the non-minimal coupling between the Einstein-Hilbert action and the scalar field vanishes in the action (6). Furthermore, by redefining the variable as $\varphi(x) \equiv \sqrt{6}\sigma(x)/\kappa$, the kinetic term of the scalar field is canonically normalized. We then find the $F(R)$ gravity is expressed as the general relativity minimally coupling with the scalar field:

$$\begin{aligned} S = & \frac{1}{2\kappa^2} \int d^4x \sqrt{-\tilde{g}} \tilde{R} + \int d^4x \sqrt{-\tilde{g}} \left[-\frac{1}{2} \tilde{g}^{\mu\nu} (\partial_\mu \varphi) (\partial_\nu \varphi) \right. \\ & \left. - \frac{1}{2\kappa^2} \frac{F_A(A(\varphi))A(\varphi) - F(A(\varphi))}{F_A^2(A(\varphi))} \right]. \end{aligned} \quad (7)$$

The gravitational theory described as in the action (7) is called the scalar-tensor theory: the scalar field A acts as the gravitational force besides the tensor field $g_{\mu\nu}$.

Finally, we consider the effect of the scalar field to the matter sector. According to the Weyl transformation, the matter sector is expressed as

$$\begin{aligned} S_{\text{Matter}} = & \int d^4x \sqrt{-\tilde{g}} e^{-4\sqrt{1/6}\kappa\varphi} \\ & \times \mathcal{L}(g^{\mu\nu} = e^{2\sqrt{1/6}\kappa\varphi} \tilde{g}^{\mu\nu}, \Psi). \end{aligned} \quad (8)$$

Here, \mathcal{L} is the matter Lagrangian density and Ψ denotes the matter fields. After the Weyl transformation, the “dilaton” coupling of the scalar field φ to the matter fields Ψ shows up in (8). This scalar field propagates between the matter fields besides the graviton. Hereafter, we refer to this scalar field φ as “scalaron” in order to distinguish from the other matter fields.

III. $F(R)$ GRAVITY FOR THE DARK ENERGY

As we saw in the previous section, the modification of gravitational action leads to a new degree of freedom, and its couplings to the ordinary matter field are necessarily introduced. Then, the new degree of freedom modifies the gravitational interaction, which causes the different prediction from the general relativity. On the other hand, in order to be a gravitational theory, the modified gravity should explain or satisfy the astrophysical observations and constraints. In this section, we give a brief review about the requirements to avoid the Solar System constraint.

A. Scalaron potential and mass

Constraints from the violation of the equivalence principle in the Solar System often exclude modifications of gravity although the modifications are required for the DE in the cosmological scale. Thus, we need to suppress the fifth force mediated by the new degree of freedom only in the smaller scale. Viable models of $F(R)$ gravity have a screening mechanism to screen the fifth force mediated by the scalaron and avoid the constraint, which is called the chameleon mechanism. In this subsection, we review the chameleon mechanism in the $F(R)$ gravity.

We first consider the equation of motion for the scalaron field. By variation of (7) with respect to the scalaron field φ , we obtain the equation of motion of the scalaron

$$\tilde{\square}\varphi = V'_{\text{eff}}(\varphi),$$

where the effective potential $V_{\text{eff}}(\varphi)$ is defined as

$$V'_{\text{eff}}(\varphi) \equiv V'(\varphi) + \frac{\kappa}{\sqrt{6}} e^{-4\sqrt{1/6}\kappa\varphi} T^\mu_\mu. \quad (9)$$

We find that the scalaron couples to the trace of the energy-momentum tensor in the equation of motion. For simplicity, we consider the non-relativistic perfect fluid with the constant energy density in the Jordan frame. The energy-momentum tensor is then expressed as $T_{\mu\nu} = \text{diag}[\rho, 0, 0, 0]$, and $T^\mu_\mu = g^{\mu\nu}T_{\mu\nu} = -\rho$. In this case, the effective potential (9) is given by

$$V_{\text{eff}}(\varphi) = V(\varphi) + \frac{1}{4} e^{-4\sqrt{1/6}\kappa\varphi} \rho. \quad (10)$$

If $V'(\varphi) > 0$, the potential has a minimum at $\varphi = \varphi_{\text{min}}$. Performing the Taylor expansion of the effective potential around $\varphi = \varphi_{\text{min}}$, we find

$$V_{\text{eff}}(\varphi) = V_{\text{eff}}(\varphi_{\text{min}}) + \frac{1}{2} V''_{\text{eff}}(\varphi_{\text{min}}) (\varphi - \varphi_{\text{min}})^2 + \dots \quad (11)$$

Here, the mass of the scalaron field is defined as the coefficient of $(\varphi - \varphi_{\text{min}})^2$ in Eq. (11), and we find

$$m_\varphi^2 \equiv V''_{\text{eff}}(\varphi_{\text{min}}) = V''(\varphi_{\text{min}}) + \frac{2\kappa^2}{3} e^{-4\sqrt{1/6}\kappa\varphi_{\text{min}}} \rho. \quad (12)$$

Therefore, if ρ is larger, the scalaron becomes heavier: in the bulk of the Universe, where the energy density is very small, the scalaron can be very light and produce the effective cosmological constant. On the other hand, in or around the heavy objects, the Solar System or the Earth, the scalaron becomes heavy. Then, the Compton wavelength becomes short and the scalaron is screened.

B. Starobinsky model

In the previous subsection, we discussed the chameleon mechanism in the $F(R)$ gravity without specifying the function of $F(R)$. In this subsection, we consider, for example, the Starobinsky model [25] for the late-time acceleration, and see how the chameleon mechanism works. The action of the Starobinsky model is defined as

$$F(R) = R - \beta R_c \left[1 - \left(1 + \frac{R^2}{R_c^2} \right)^{-n} \right]$$

with constants n , β , and $R_c > 0$. In the limit $R/R_c \gg 1$, we find

$$F(R) \approx R - \beta R_c + \beta R_c \left(\frac{R}{R_c} \right)^{-2n}. \quad (13)$$

If we ignore the third term in the large curvature regime, the Starobinsky model restores the GR with the cosmological constant, $\Lambda = \beta R_c/2$. In this subsection, we study the approximated model (13) because the chameleon mechanism works in the large curvature regime $R \gg R_c$.

First, we calculate the scalaron potential. From the definition of the Weyl transformation, we find $e^{2\sqrt{1/6}\kappa\varphi} = 1 - 2n\beta (R/R_c)^{-(2n+1)}$. Here, we note that the scalaron field φ is negative, and it goes to zero $\varphi \rightarrow 0 - 0$ as the curvature R increases. Assuming the curvature is larger than the cosmological constant, $R \gg R_0 \sim R_c$, we find

$$|\kappa\varphi| = \frac{\sqrt{6}}{2} \left| \ln \left(1 - 2n\beta \left(\frac{R}{R_c} \right)^{-(2n+1)} \right) \right| \ll 1. \quad (14)$$

Here, we note that $2n\beta (R/R_c)^{-(2n+1)} < 1$ because $F_R(R) > 0$: for the consistency with the observation,

it is required for $F(R)$ gravity models to avoid the anti-gravity. If $F_R(R) < 0$, the coefficient in front of the Einstein-Hilbert part in (6) is negative. In other words, the gravitational constant becomes negative, which leads to the anti-gravity.

When we consider the non-relativistic matter for $T_{\mu\nu}$, the scalaron potential is given by

$$V_{\text{eff}}(\varphi) = \frac{\beta R_c}{2\kappa^2} e^{-4\sqrt{1/6}\kappa\varphi} \times \left[1 - (2n+1) \left\{ \frac{1}{2n\beta} \left(1 - e^{2\sqrt{1/6}\kappa\varphi} \right) \right\}^{\frac{2n}{2n+1}} \right] + \frac{1}{4} e^{-4\sqrt{1/6}\kappa\varphi} \rho. \quad (15)$$

Here, we note that the chameleon mechanism works at the high density region where the curvature should be large. So, in the following calculation, we assume $|\kappa\varphi| \ll 1$ as in (14). In this limit, the effective potential (15) is approximated to be

$$V_{\text{eff}}(\varphi) \approx \frac{\beta R_c}{2\kappa^2} e^{-4\sqrt{1/6}\kappa\varphi} \times \left[1 - (2n+1) \left(-\frac{\kappa\varphi}{\sqrt{6}n\beta} \right)^{\frac{2n}{2n+1}} + \frac{\kappa^2\rho}{2\beta R_c} \right]. \quad (16)$$

Second, we study the minimum of the effective potential. The derivative of the effective potential (16) is

$$V'_{\text{eff}}(\varphi) \approx -\frac{4\kappa}{\sqrt{6}} \frac{\beta R_c}{2\kappa^2} e^{-4\sqrt{1/6}\kappa\varphi} \times \left[1 - (2n+1) \left(-\frac{\kappa\varphi}{\sqrt{6}n\beta} \right)^{\frac{2n}{2n+1}} - \frac{1}{2\beta} \left(-\frac{\kappa\varphi}{\sqrt{6}n\beta} \right)^{-\frac{1}{2n+1}} + \frac{\kappa^2\rho}{2\beta R_c} \right]. \quad (17)$$

Solving $V'_{\text{eff}} = 0$ in (17), we find the minimum $\varphi = \varphi_{\text{min}}$,

$$\kappa\varphi_{\text{min}} = -\sqrt{6}n\beta \left(\frac{R_c}{\kappa^2\rho} \right)^{\frac{2n+1}{2n}}. \quad (18)$$

Finally, we evaluate the scalaron mass. The second derivative of the effective potential (16) is calculated as

$$V''_{\text{eff}}(\varphi) \approx \left(\frac{4\kappa}{\sqrt{6}} \right)^2 \frac{\beta R_c}{2\kappa^2} e^{-4\sqrt{1/6}\kappa\varphi} \times \left[1 - (2n+1) \left(-\frac{\kappa\varphi}{\sqrt{6}n\beta} \right)^{\frac{2n}{2n+1}} + \frac{\kappa^2\rho}{2\beta R_c} - \frac{1}{\beta} \left(-\frac{\kappa\varphi}{\sqrt{6}n\beta} \right)^{-\frac{1}{2n+1}} + \frac{1}{8n(2n+1)\beta^2} \left(-\frac{\kappa\varphi}{\sqrt{6}n\beta} \right)^{-\frac{1}{2n+1}-1} \right]. \quad (19)$$

Substituting Eq. (18) into Eq. (19) with $\varphi = \varphi_{\text{min}}$, we obtain the expression of the scalaron mass

$$m_\varphi^2 \approx \frac{R_c}{6n(2n+1)\beta} \left(\frac{\kappa^2\rho}{R_c} \right)^{2(n+1)}. \quad (20)$$

We note that the scalaron mass depends on the energy density, and increases like a power function of ρ .

IV. $F(R)$ GRAVITY FOR DARK MATTER: SCALARON AS DARK MATTER

We have seen how the chameleon mechanism works in the $F(R)$ gravity theory. In this section, we reconsider the new coupling between the scalaron φ and the SM particles in the Einstein frame. It may bring us a fascinating fact that the new massive scalar field is naturally

derived from the modification of gravity without extending the particle contents of the SM. In other words, the modification of gravitational theory affects the particle physics, and the phenomena beyond the SM may show up.

We recall the properties of the scalaron as we have seen so far: (1) after the Weyl transformation, dilatonic interactions between the scalaron and the SM appear. The coupling is suppressed in the large curvature regime; (2) the scalaron mass is very large in the large curvature regime because of the chameleon mechanism. As a result, the fifth force, the propagation of the scalaron, is suppressed, and it is consistent with the observational constraint. These two natures of the scalaron imply that a massive field weakly coupled with SM particles emerges in the Solar System, or around the Earth. Therefore, this property suggests that the scalaron could be a CDM. In other words, the ‘‘Darkness’’ of DM is justified by the

dilatonic coupling with the Planck mass suppression and the chameleon mechanism in this scenario.

In order to study the scalaron field as a DM candidate, it is necessary to formulate the couplings between the scalaron and the SM particles. In this section, we investigate the matter coupling in Eq. (8) and determine the form and magnitude of couplings.

A. Coupling to massless fields

First, we consider the massless vector field A_μ . The Lagrangian density \mathcal{L}_V is given by

$$\mathcal{L}_V(g^{\mu\nu}, A_\mu) = -\frac{1}{4}e^{4\sqrt{1/6}\kappa\varphi}\tilde{g}^{\alpha\mu}\tilde{g}^{\beta\nu}F_{\alpha\beta}F_{\mu\nu}. \quad (21)$$

For simplicity, we consider an abelian gauge field. The generalization to the curved space-time can be made by replacing the partial derivative with the covariant derivative in the field strength $F_{\mu\nu}$. However, we find that the field strength does not change because the Christoffel symbols are symmetric, and $F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$ even in the curved space-time. Thus, the field strength $F_{\mu\nu}$ is invariant under the Weyl transformation, so that the action in the Einstein frame is given just by the replacing $g^{\mu\nu} \rightarrow \tilde{g}^{\mu\nu}$:

$$S = \int d^4x \sqrt{-\tilde{g}} \mathcal{L}_V(\tilde{g}^{\mu\nu}, A_\mu), \quad (22)$$

where the exponential factor $e^{4\sqrt{1/6}\kappa\varphi}$ in (21) has been canceled with $e^{-4\sqrt{1/6}\kappa\varphi}$ in (8). We find that the direct coupling between the massless vector field A_μ and the scalaron φ does not arise through the field strength. This feature is in sharp contrast to the dilatonic dark matter although the coupling form appears to be similar as in (22). The same argument is applicable even for the non-abelian gauge field.

Second, we consider the massless fermion $\psi(x)$. Unlike the case of bosonic fields, we need special treatment for fermion fields. The Lagrangian density \mathcal{L}_F in curved space-time is given by

$$\mathcal{L}_F(\gamma^\mu, \psi) = i\bar{\psi}(x)\gamma^\mu\nabla_\mu\psi(x). \quad (23)$$

γ^μ is the generalized Dirac gamma matrix in curved space-time, defined as $\gamma^\mu(x) \equiv e_a^\mu(x)\gamma^a$, where $\{\gamma^\mu, \gamma^\nu\} = 2g^{\mu\nu}$. And, the vierbein e_μ^a is related to the metric as $g_{\mu\nu}(x) = \eta_{ab}e_\mu^a(x)e_\nu^b(x)$ with Latin letters a, b, \dots for Lorentz indices, and Greek letters μ, ν, \dots for space-time indices. The covariant derivative for the spinor is given by

$$\nabla_\mu\psi(x) = \partial_\mu\psi(x) + \frac{1}{8}\omega_{\mu ab}(x)[\gamma^a, \gamma^b]\psi(x),$$

where $\omega_{\mu ab}(x)$ is called a spin connection, defined as $w_{\mu ab}(x) = e_{a\nu}(\partial_\mu e_b^\nu + \Gamma_{\mu\rho}^\nu e_b^\rho)$. Under the Weyl transformation $\tilde{g}_{\mu\nu} = e^{2\sigma}g_{\mu\nu}$, the vierbein e_μ^a and the generalized Dirac gamma matrix γ^μ transform as $\tilde{e}_\mu^a = e^\sigma e_\mu^a$

and $\tilde{\gamma}^\mu = e^{-\sigma}\gamma^\mu$, respectively. Hence, the spin connection transforms as $\omega_{\mu ab} = \tilde{\omega}_{\mu ab} - (\tilde{e}_{a\mu}\tilde{e}_b^\lambda - \tilde{e}_{b\mu}\tilde{e}_a^\lambda)\partial_\lambda\sigma$.

Using those transformation rules, we find that the Lagrangian density (23) in the Einstein frame is given by

$$\begin{aligned} \mathcal{L}_F(\gamma^\mu, \psi) = & e^{\sqrt{1/6}\kappa\varphi} i\bar{\psi}\tilde{\gamma}^\mu\tilde{\nabla}_\mu\psi \\ & - \frac{3i}{2}\sqrt{\frac{1}{6}}\kappa e^{\sqrt{1/6}\kappa\varphi}(\partial_\mu\varphi)\bar{\psi}\tilde{\gamma}^\mu\psi, \end{aligned}$$

where we used $\sigma = \sqrt{1/6}\kappa\varphi$. Then, the action in the Einstein frame takes the following form:

$$\begin{aligned} S = & \int d^4x \sqrt{-\tilde{g}} e^{-4\sqrt{1/6}\kappa\varphi} \mathcal{L}_F(\gamma^\mu, \psi) \\ = & \int d^4x \sqrt{-\tilde{g}} \left[e^{-3\sqrt{1/6}\kappa\varphi} i\bar{\psi}\tilde{\gamma}^\mu\tilde{\nabla}_\mu\psi \right. \\ & \left. - \frac{3i}{2}\sqrt{\frac{1}{6}}\kappa e^{-3\sqrt{1/6}\kappa\varphi}(\partial_\mu\varphi)\bar{\psi}\tilde{\gamma}^\mu\psi \right]. \quad (24) \end{aligned}$$

Note that the couplings between the scalar field ψ and scalaron φ are generated in (24) when $\kappa\varphi \neq 0$, in contrast to the case of massless vector fields.

Now, we divide the action into two parts:

$$S = \int d^4x \sqrt{-\tilde{g}} [\mathcal{L}_F(\tilde{\gamma}^\mu, \psi) + \mathcal{L}_{F-\varphi}(\tilde{\gamma}^\mu, \psi, \varphi)], \quad (25)$$

where

$$\begin{aligned} \mathcal{L}_{F-\varphi} \equiv & (e^{-3\sqrt{1/6}\kappa\varphi} - 1) i\bar{\psi}\tilde{\gamma}^\mu\tilde{\nabla}_\mu\psi \\ & - \frac{3i}{2}\sqrt{\frac{1}{6}}\kappa e^{-3\sqrt{1/6}\kappa\varphi}(\partial_\mu\varphi)\bar{\psi}\tilde{\gamma}^\mu\psi. \quad (26) \end{aligned}$$

The first term in (25) is the action of the fermion field in the Einstein frame where the metric is replaced as $g_{\mu\nu} \rightarrow \tilde{g}_{\mu\nu}$. The second term describes the non-linear interaction between the fermion field ψ and scalaron φ .

As done in Eq. (14), we consider the weak coupling limit $\kappa\varphi \ll 1$, corresponding to the large curvature limit $R \gg R_0$. In this limit, we can expand the dilatonic coupling $e^{\kappa\varphi}$ in the Lagrangian density (26), and obtain

$$\begin{aligned} \mathcal{L}_{F-\varphi}(\tilde{\gamma}^\mu, \psi, \varphi) = & -\frac{3i\kappa}{\sqrt{6}}\varphi\bar{\psi}\tilde{\gamma}^\mu\tilde{\nabla}_\mu\psi - \frac{3i\kappa}{2\sqrt{6}}(\partial_\mu\varphi)\bar{\psi}\tilde{\gamma}^\mu\psi \\ & + \mathcal{O}(\kappa^2\varphi^2). \quad (27) \end{aligned}$$

We note that the couplings between the vector field A_μ and the scalaron φ arise in (27) when we consider the gauge covariant derivative. By replacing

$$\tilde{\nabla}_\mu \rightarrow \tilde{\nabla}_\mu - igA_\mu,$$

where g is the gauge coupling constant, and the coupling takes the bilinear form of fermion.

B. Coupling to massive fields

Finally, we consider the massive vector and fermion fields. After the electroweak symmetry breaking, the vector and fermion fields acquire the mass through the Higgs mechanism. The mass term of the vector field $\mathcal{L}_{V-\text{mass}}$ is given by

$$\mathcal{L}_{V-\text{mass}}(g^{\mu\nu}, A_\mu) = -\frac{1}{2}m_V^2 g^{\mu\nu} A_\mu A_\nu,$$

where m_V is the mass of the massive vector field. In a way similar to the massless fermion case, the action in the Einstein frame can be divided into two parts:

$$\begin{aligned} S &= \int d^4x \sqrt{-\tilde{g}} e^{-4\sqrt{1/6}\kappa\varphi} \mathcal{L}_{V-\text{mass}}(g^{\mu\nu}, A_\mu) \\ &= \int d^4x \sqrt{-\tilde{g}} [\mathcal{L}_{V-\text{mass}}(\tilde{g}^{\mu\nu}, A_\mu) \\ &\quad + \mathcal{L}_{V-\varphi}(\tilde{g}^{\mu\nu}, A_\mu, \varphi)], \end{aligned}$$

where

$$\begin{aligned} \mathcal{L}_{V-\varphi}(\tilde{g}^{\mu\nu}, A_\mu, \varphi) \\ \equiv -\frac{1}{2}m_V^2 \left(e^{-2\sqrt{1/6}\kappa\varphi} - 1 \right) \tilde{g}^{\mu\nu} A_\mu A_\nu. \end{aligned} \quad (28)$$

Expanding the Lagrangian density (28) with respect to $|\kappa\varphi| \ll 1$, we find

$$\mathcal{L}_{V-\varphi}(\tilde{g}^{\mu\nu}, A_\mu, \varphi) = \frac{\kappa m_V^2}{\sqrt{6}} \varphi \tilde{g}^{\mu\nu} A_\mu A_\nu + \mathcal{O}(\kappa^2 \varphi^2). \quad (29)$$

Thus, we have the couplings between the massive vector field A_μ and the scalaron φ .

The mass term of the fermion field is given by

$$\mathcal{L}_{F-\text{mass}}(\psi) = -m_F \bar{\psi} \psi,$$

where m_F is the mass of the massive fermionic field. As in the case of the massive vector field, we obtain

$$S = \int d^4x \sqrt{-\tilde{g}} [\mathcal{L}_{F-\text{mass}}(\psi) + \mathcal{L}_{F-\varphi}(\psi, \varphi)],$$

where

$$\mathcal{L}_{F-\varphi}(\psi, \varphi) \equiv -m_F \left(e^{-4\sqrt{1/6}\kappa\varphi} - 1 \right) \bar{\psi} \psi. \quad (30)$$

Expanding the Lagrangian density (30) with respect to $|\kappa\varphi| \ll 1$, we find

$$\mathcal{L}_{F-\varphi}(\psi, \varphi) = \frac{4\kappa m_F}{\sqrt{6}} \varphi \bar{\psi} \psi + \mathcal{O}(\kappa^2 \varphi^2). \quad (31)$$

Again, we have the couplings between the massive fermion field A_μ and the scalaron φ .

V. AN EFFECTIVE MODEL FOR SCALARON PARTICLE

In the previous section, the interactions between the scalaron and the SM particles have been investigated. However, we need to discuss the particle picture of the scalaron field although we usually study the classical dynamics of the scalaron field as a gravitational theory.

A. Particle picture of the scalaron

We shall first expand the scalaron field φ around the background solution $\varphi = \varphi_{\min}$: $\varphi = \hat{\varphi} + \varphi_{\min}$, and treat the fluctuation $\hat{\varphi}$ as a particle. We now consider the role of the chameleon mechanism for the background solution φ_{\min} and the scalaron “particle” $\hat{\varphi}$, respectively.

As we discussed in Eq. (10), the scalaron potential changes through the chameleon mechanism according to the trace of the energy-momentum tensor. Hence, the chameleon mechanism reflects the environment dependence of the scalaron: the energy-momentum tensor consists of the matter fields surrounding the scalaron, which controls the environment system. In order to take the chameleon mechanism into account, we need to specify the energy-momentum tensor corresponding to the SM environment. In the present analysis, we assume that the SM bulk is described by the perfect fluid, and the energy density ρ is namely given as $\rho_{\text{EW}} \sim (100 \text{ GeV})^4$ in Eq. (10).

Thus, the explicit environment dependence enters in the background solution φ_{\min} through the equation of motion,

$$\hat{\square} \varphi_{\min} = V'(\varphi_{\min}) - \frac{\kappa}{\sqrt{6}} e^{-4\sqrt{1/6}\kappa\varphi_{\min}} \rho_{\text{EW}}.$$

As for the scalaron particle $\hat{\varphi}$, the environment dependence implicitly arises in the mass expression, which is given by substituting $\rho = \rho_{\text{EW}}$ into Eq. (12),

$$m_{\hat{\varphi}}^2 = V''(\varphi_{\min}) + \frac{2\kappa^2}{3} e^{-4\sqrt{1/6}\kappa\varphi_{\min}} \rho_{\text{EW}}. \quad (32)$$

Performing the fluctuation around the background solution, we find

$$\begin{aligned} S_{\text{Matter}} &= \int d^4x \sqrt{-\tilde{g}} e^{-4\sqrt{1/6}\kappa(\varphi_{\min} + \hat{\varphi})} \\ &\quad \times \mathcal{L}_{\text{SM}} \left(e^{2\sqrt{1/6}\kappa(\varphi_{\min} + \hat{\varphi})} \tilde{g}^{\mu\nu}, \Psi \right). \end{aligned}$$

Because $|\kappa\varphi_{\min}| \ll 1$, the scalaron coupling to the SM is approximately given by

$$\begin{aligned} S_{\text{Matter}} &\approx \int d^4x \sqrt{-\tilde{g}} e^{-4\sqrt{1/6}\kappa\hat{\varphi}} \\ &\quad \times \mathcal{L}_{\text{SM}} \left(e^{2\sqrt{1/6}\kappa\hat{\varphi}} \tilde{g}^{\mu\nu}, \Psi \right), \end{aligned} \quad (33)$$

so that we utilize the result in the previous section just by replacing $\varphi \rightarrow \hat{\varphi}$. The environment dependence still implicitly remains in the scalaron mass in (32) while the effect of background solution in the scalaron coupling has been ignored in (33). Note that we are considering the microscopic environment where the scalaron is touching with the SM particles. When one discusses the chameleon mechanism in the Solar System, the averaged energy density $\rho_\odot \sim 10 \text{ g/cm}^3 = 10^{19} \text{ eV}^4$, which implies ρ_{EW} is large enough to cause the chameleon mechanism even in the microscopic environment.

B. Lifetime of scalaron and constraint to parameters in $F(R)$ models

Because the scalaron potential depends on the function of $F(R)$ as well as the environment, the constraint as a DM candidate can be rephrased as the constraint on the form of the $F(R)$ function. In this subsection, we study the decay process of the scalaron, evaluate the lifetime, and give the constraint on the parameter in the Starobinsky model.

As we have seen in the previous section, the scalaron does not directly couple to massless vector fields because of the Weyl transformation invariance. The massless fermion fields have the coupling to the scalaron, which is, however, small because of the derivative coupling carrying the momentum on the same order of the fermion mass. Thus, we may pay a particular attention to the scalaron coupling with massive fields in the SM, as in the Lagrangians (29) and (31). Such a coupling strength is essentially set by the mass of the field coupled to the scalaron, just like the Higgs boson or the dilaton. Therefore, one might think that the decay modes to the top quark and W boson pairs are dominant. However, such a heavy scalaron cannot live longer than the age of the Universe, so cannot be a DM candidate.

In order to be a DM candidate, the scalaron has to be so light that it cannot decay to the massive particles. In that case, the dominant contributions of scalaron decay come from the coupling to diphoton through the top quark and W bosons loops. Then, the decay width can be evaluated at the one-loop level of the electroweak-perturbation theory:

$$\Gamma_{\hat{\varphi} \rightarrow \gamma\gamma} \approx \frac{\alpha_{\text{em}}}{256\pi^3} \frac{m_{\hat{\varphi}}^3}{M_{\text{pl}}^2} \left| 3 \cdot \left(\frac{2}{3}\right)^2 \cdot \left(\frac{4}{3}\right) + (-7) \right|^2, \quad (34)$$

where α_{em} denotes the fine-structure constant, the prefactor 3 counts the number of colors carried by the top quark, and the factor $(2/3)$ stands for the electromagnetic charge of the top quark. In reaching Eq. (34), we have taken the infinite-mass limit for the top quark and W boson, which reflects the factors $(4/3)$ for the top quark, and (-7) for W boson. Requiring that the lifetime is longer than the age of the Universe, $\Gamma_{\hat{\varphi} \rightarrow \gamma\gamma}^{-1} > 10^{17} \text{ s}$, we find the upper bound for the scalaron mass as

follows:

$$m_{\hat{\varphi}} \lesssim 1 \text{ GeV}. \quad (35)$$

Finally, we convert the mass bound (35) into the constraint on the form of the $F(R)$ function. For the Starobinsky model with $\beta R_c = 2\Lambda$, the scalaron mass is given by substituting $\rho = \rho_{\text{EW}}$ into Eq. (20):

$$m_{\hat{\varphi}}^2 \approx \frac{2\Lambda}{6n(2n+1)\beta^2} \left(\frac{\kappa^2 \beta}{2\Lambda} \rho_{\text{EW}} \right)^{2(n+1)},$$

which leads to the constraint on the parameter β : $\beta \lesssim 10^{-68}$ for $n = 1$, and $\beta \lesssim 10^{-59}$ for $n = 4$. We note that the constraint obtained here has been estimated in the Einstein frame. One would obtain almost the same result in magnitude even in the Jordan frame because the Weyl transformation operates as almost identity $e^{2\sigma} \sim 1$, i.e., $\tilde{g}_{\mu\nu} \sim g_{\mu\nu}$.

VI. SUMMARY AND DISCUSSION

We have studied the scalaron field in the $F(R)$ gravity from the viewpoint of particle physics. We have assumed that the scalaron is a DM candidate, and evaluated its lifetime from the decay to diphoton. We have placed the constraint on the scalaron mass, and obtained the constraint on the parameter β in the Starobinsky model. We have shown that β should be extremely small for the scalaron to be a DM although it is desired to be $\mathcal{O}(1)$ for the modified gravity to be a solution for the DE. This discrepancy naively implies an answer to the question raised in the abstract: the scalaron DM scenario may be incompatible with the DE problem.

However, we are still left with open questions to avoid the incompatibility. We should be even skeptical about our result because some assumptions were made to derive the constraint on the parameter β . Hereafter, we shall reconsider the prescriptions in the analysis, and discuss the possible three ways to improve the gigantic suppression factor for the parameter β .

First, we shall consider the validity of the particle picture of the scalaron. We have studied the microscopic nature of the scalaron field by using the quantum field theory, and derived the constraint on the modified gravity through the upper limit of the scalaron mass. Here, the result implies that the scalaron mass should be extremely heavy if we require $\beta = \mathcal{O}(1)$. In this case, the scalaron cannot be handled with the scalaron field as a particle, thus, it should be treated in classical way. If the scalaron is a classical object, we need other prescriptions to discuss the stability of scalaron, and one can expect it is stable and does not decay.

Second, we shall reconsider the energy-momentum tensor of the environment system surrounding the scalaron. When we estimate the scalaron mass, the choice of energy-momentum tensor plays a crucial role because the

effective potential is sensitive to the environment according to the chameleon mechanism. In our analysis, we assumed that background matter field is expressed as a perfect fluid, which is the coarse-grained picture of the matter field. Here, we assume the realistic matters consist of particles, that is, an atom or a molecule. If the scalaron is heavy, its Compton wavelength is short, and the scalaron cannot touch the particles. On the other hand, the environment between the particles is almost like a vacuum, and the Compton wavelength becomes large because the chameleon mechanism no longer works. Thus, we may expect that the Compton wavelength of the scalaron should be comparable with the coarse-graining scale, that is, interatomic distance, which means the scalaron cannot be heavy.

Third, we shall discuss the model of $F(R)$ gravity. If we admit the constraint on the parameter β , we need to construct a new model of $F(R)$ gravity, where the light scalaron field is realized even in the high density region. From the viewpoint of cosmology, the fifth force should be suppressed and the heavier scalaron is preferred. On the other hand, the scalaron cannot be too heavy for a DM candidate. Thus, if we can improve the models of

$F(R)$ gravity for the DE and the DM, the compatibility to observations and experiments for the fifth force gives a constraint on the $F(R)$ gravity.

Because of the above reasons, it may be premature to conclude that the scalaron cannot be a DM candidate. These open questions will be treated in the future works. Besides, we will also study and improve the methods to evaluate the thermal history and relic abundance of the scalaron [26, 27].

Our analysis in this paper can be applied for other modified gravity theories, which would give more various DM candidates.

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